

# 1. Introduction to Haskell

1.1. Main constructs of Haskell

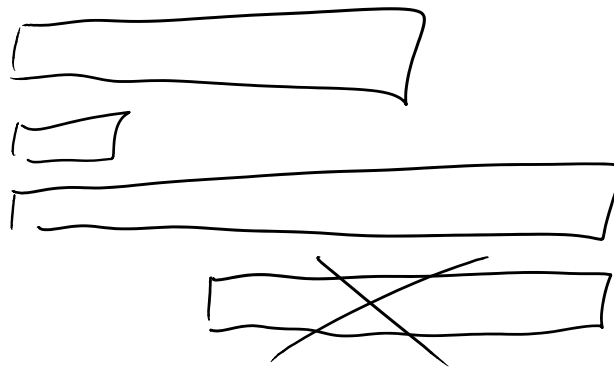
1.2. } Prog. techniques in functional languages  
1.3 }  
1.4 }

## 1.1. Main constructs of Haskell

4 kinds of main constructs: declarations, expressions, patterns, types

### 1.1.1. Declarations

Haskell prog = sequence of declarations that have to start at the same column



Declarations are either type declarations or function declarations

we define Haskell's syntax by EBNF-rules where non-terminal symbols are underlined

## Comments in Haskell:

{-

:

-}

or

--

... until  
end of line

## Type declarations (Slide 4)

square :: Int → Int

↑

Variables are used  
as names of functions

↑ Type of Functions from Int  
to Int

Haskell has pre-defined types  
like Int, Bool, ...

One can create new types by  
"→" or by "[... ]":

Int → Bool

[Int]

[Int] → Int

Type declarations do not have to be specified by the programmer (then Haskell computes them automatically).

But it is good programming style to provide them.

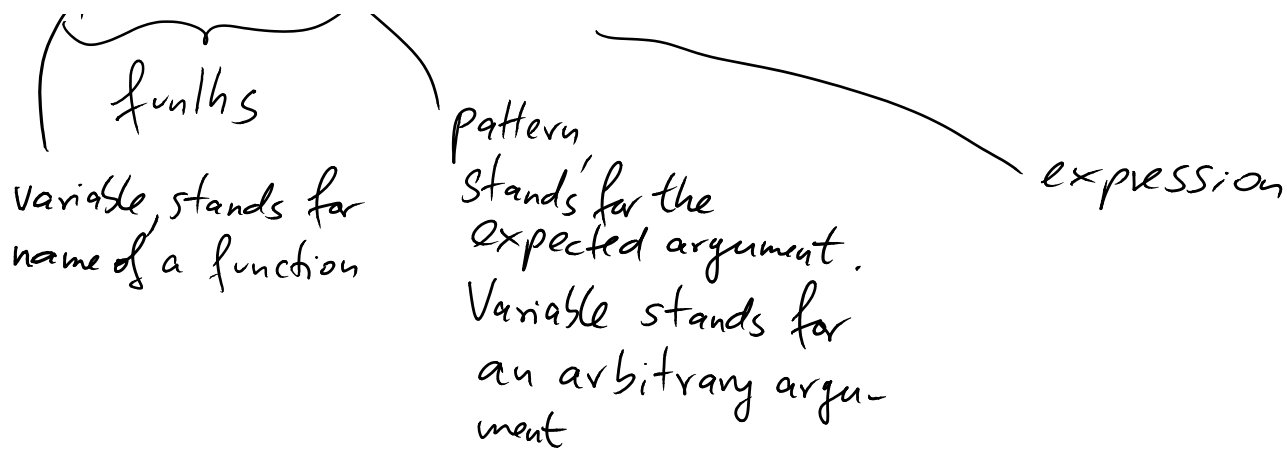
## Function declarations

square x = x \* x

↑  
lunhc

↑  
..

Defining equations  
specifying a function



Haskell has certain built-in operations:

arithmetic operations:  $+$ ,  $-$ ,  $*$ ,  $/$ , ...

comparison operations:  $==$ ,  $>$ ,  $<$ ,  $>=$ ,  $<=$ , ...

Boolean operations: `not`, `&&`, `||`, ...

→ they evaluate to True or False

## Evaluation of functional programs (Slide 5)

Program tries to simplify a given expression.

Ex: `Square (12 - 1)`

Evaluation works by term rewriting:

1. Computer searches for a sub-expression such that the left-hand side of a defining equation matches this sub-expression (i.e., the lhs is equal to the sub-expression if one instantiates its variables appropriately).
2. Replace the instantiated lhs of the equation

by the instantiated right-hand side.

$Square :: Int \rightarrow Int$

$Square\ x = x * x$

2 main evaluation strategies:

- leftmost innermost (call-by-value)  
disadvantage: one evaluates all subexpressions, even if they are not needed for the result  $\rightarrow$  might lead to (unnecessary non-termination)
- leftmost outermost (call-by-name)  
disadvantage: duplicated arguments have to be evaluated several times

Haskell: Lazy Evaluation

leftmost outermost, but Haskell keeps track of duplicated arguments and only evaluates them once in parallel.

$three :: Int \rightarrow Int$

$three\ x = 3$

$non\_term :: Int \rightarrow Int$

$non\_term\ x = non\_term\ (x+1)$

$non\_term\ 0$  does not terminate

$three\ (non\_term\ 0)$  terminates in 1 evaluation step, result is 3

Haskell implementation



We recommend the GHC (Glasgow Haskell Compiler) in particular the  $GHC_i$  interpreter.  
↑  
"interactive"

## Conditional equations (Slide 6)

$maxi :: (Int, Int) \rightarrow Int$

$maxi (x,y) \mid x \geq y = x$

$\mid otherwise = y$

$(Int, Int)$ : type of tuples of 2 components, both have type  $Int$ .

$(2,3)$  has type  $(Int, Int)$

- 
- rhs of an equation can be restricted by a Boolean condition
- Conditions are checked from top to bottom, don't have to be exhaustive
- "otherwise" always evaluates to True

## Currying

named after logician Haskell B. Curry

Idea: don't provide several arguments as a tuple, but provide them individually one after another

$plus :: Int \rightarrow (Int \rightarrow Int)$

$plus\ x\ y = x + y$

$plus\ 2$   
is a function of type  $Int \rightarrow Int$

Here:

$(plus\ 2)\ 3 = 5$

$\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$  stands for  $\text{Int} \rightarrow (\text{Int} \rightarrow \text{Int})$   $\rightarrow$  associates to the right

plus x y stands for (plus x) y

function application associates to the left

Advantages of Currying:

- less brackets
- "partial applications" are possible: one may apply a function to fewer arguments

plus 2 is an expression of type  $\text{Int} \rightarrow \text{Int}$ .

It stands for the function that takes an argument and increases it by 2.

$\Rightarrow$  var can now be applied to several arguments  $\text{pat}_1, \dots, \text{pat}_n$  (Slide 7).

## Function definition by pattern matching

$\text{und} :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$   
 $\text{und True } y = y$   
 $\text{und False } y = \text{False}$

2 fct. declarations for the same fct. symbol ("und").

Patterns (True, False, y) are used to describe the expected arguments.

Data type Bool has 2 data constructors: True, False

Such functions are used to construct data objects, can't be evaluated further.

Pattern matching: perform a case analysis depending on the data constructors that the arguments are built with.

To evaluate  $\text{und } \underline{\text{exp}_1} \ \underline{\text{exp}_2}$   
one first has to check, whether the lhs of the first equation matches  $\text{und } \underline{\text{exp}_1} \ \underline{\text{exp}_2}$ .

If yes: use 1st equation for evaluation

If no: check the next equation

↳ equations are checked from top to bottom

↳ one could simplify "und" to:

$\text{und} :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$

$\text{und True } y = y$

$\text{und } x \ y = \text{False}$

$\text{unclear} :: \text{Int} \rightarrow \text{Bool}$

$\text{unclear } x = \text{not } (\text{unclear } x)$

$\text{und False } (\text{unclear } 0) = \text{False}$

$\text{und } (\text{unclear } 0) \ \text{False}$  does not terminate:

Haskell needs to find out whether the pattern `True` matches `(unclear 0)`. To this end, it evaluates `(unclear 0)` step by step and checks whether `True` matches the result ...

Pattern matching for the datatype `Bool`:

Bool  $\rightarrow$  `True` | `False`

Pattern matching for data type of lists (`[a]`):

$\underline{[a]} \rightarrow \underline{[]}$  |  $\underline{a} : \underline{[a]}$   
 Data constructor for empty list      Data constructor for non-empty lists

$len :: [a] \rightarrow Int$   
 $len [] = 0$   
 $len (x : xs) = 1 + len xs$

Pattern  $\hat{=}$  expression  
 built from variables  
 and data constructors (True, False, [], :, ...)

Evaluation of  $len [15, 70, 36]$   
 $15 : (70 : (36 : []))$  or  
 $15 : 70 : 36 : []$   
 $= 1 + len [70, 36]$   
 $= \dots = 3$

$\leftarrow$  : associates to the right

$second :: [Int] \rightarrow Int$

$second [] = 0$

$second (x : []) = 0$

$\leftarrow second [x] = 0$

$second (x : y : xs) = y$

Defining equations don't have to be exhaustive. (Slide 7)

Pattern declarations (Slide 8)

Can be used to declare constants (functions of arity 0)

$plus\ x\ y = x + y$

$$\text{succ } 5 = \text{plus } 1 \ 5 = 1 + 5 = 6$$

A pattern declaration assigns a certain expression to a pattern.

## Local declarations (Slide 9)

Local declarations are only valid within some other declaration.

Ex: Given  $a, b, c \in \mathbb{Q}$ , find  $x \in \mathbb{Q}$  such that

$$a x^2 + b x + c = 0 \quad (\Rightarrow)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

roots :: Float -> Float -> Float -> (Float, Float)

roots a b c = ((-b - d) / e, (-b + d) / e)

where  $d = \text{sqrt } (b * b - 4 * a * c)$

$e = 2 * a$

↑ Local declarations,  
only valid in this rhs

• more readable

• more efficient, because  $d$  and  $e$  are only evaluated once

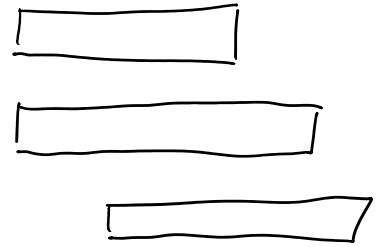
## Offside-rule for layout in Haskell

1. The first symbol in a block decls of declarations determines the leftmost column of this block.

the leftmost column of this block.

2. A new line starting in this column is a new declaration of the same block.

3. A new line starting further to the right continues the declaration in the line above.



e.g. 
$$d = \text{sqrt}(b^2 - 4 * a * c)$$

4. A new line starting further to the left means that the current declaration block has ended. The new line does not belong to this decl. block anymore.

roots  $a b c = \dots$   
where  $d = \dots$   
 $e = \dots$

Square  $x = x * x$

## Operators and infix declarations

up to now: functions in prefix notation: plus 2 3

Sometimes one wants to use infix notation instead:

$2 + 3$ ,  $2 < 3$ ,  $2 == 3$ , True || False, ...

Infix functions are also called operators.

Every infix operator can be turned into a prefix function

by using (...):  
(+) 2 3 (is 2+3)  
(:) 2 [3] (is 2:[3])

Every prefix fct. can be turned into an infix operator

by using '...': 2 'plus' 3 (is plus 2 3)

Moreover, one can also define infix operators directly.

2 properties have to be determined for infix operators:

### 1. Association

divide :: Float -> Float -> Float  
divide x y = x / y

36 'divide' 6 'divide' 2

If 'divide' associates to the left, the result is

$$(36 / 6) / 2 = 3$$

If 'divide' associates to the right, the result is

$$36 / (6 / 2) = 12$$

Haskell allows infix-declarations:

default -> infixl 'divide'  
infixr 'divide'

← this means that 'divide'  
associates to the left  
resp. to the right

infix 'divide' ← this means that 'divide' does not  
associate, i.e.,

36 'd' 6 'd' 2 is forbidden

E.g:  $\rightarrow$  associates to the right ( $\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$  stands for  $\text{Int} \rightarrow (\text{Int} \rightarrow \text{Int})$ )

function applic. associates to the left  
(plus 2 3 stands for (plus 2) 3)

square square 2 stands for  
(square square) 2  $\downarrow$  type error

## 2. Binding priority

$(\%) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$		$(@@) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
$x \% y = x + y$		$x @@ y = x * y$

What is the result of  $1 \overset{+}{\%} 2 \overset{*}{@@} 3$

One can specify binding priority by a number between 0, ..., 9:

infixl 9  $\%$

infixl 8  $@$

Then

$1 \% 2 @@ 3 =$

$(1 \% 2) @@ 3 = 9$

### Slide 10

prefix functions: strings that start with lower-case symbol  
(e.g. square)



prefix data constructors: strings that start  
with upper-case symbol (e.g. True)

infix functions: strings of special symbols not starting with :  
(e.g., +, ==, >=, @, ...)

infix data constructors: strings of special symbols starting  
with : (e.g., :)

Currying is also possible for operators:

6 `divide` has type  $\text{Float} \rightarrow \text{Float}$ .

It takes an argument  $y$  and divides 6 by  $y$ .

`divide` 6 has type  $\text{Float} \rightarrow \text{Float}$ .

It takes a number  $x$  and divides  $x$  by 6.